Supporting the improvement of the carbon observing system by quantitative network design

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Motivation

What is the question?

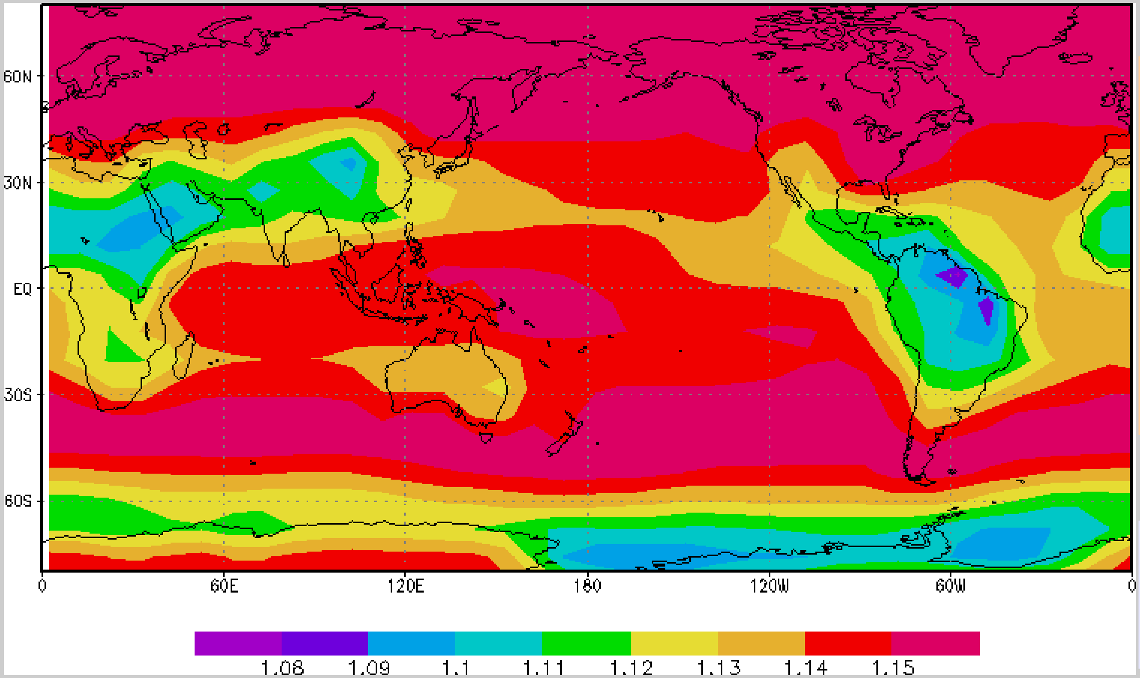
Can construct a machinery that,
for a given network and a given target quantity,
can approximate the uncertainty
with which the value of the target quantity is
constrained by the observations

Outline

- Motivation
- Example
- Method
- Demo
- Summary
- Links to further information

Example Linear Model

Target quantity:
Standard deviation of Global Ocean Uptake (GtC/yr)



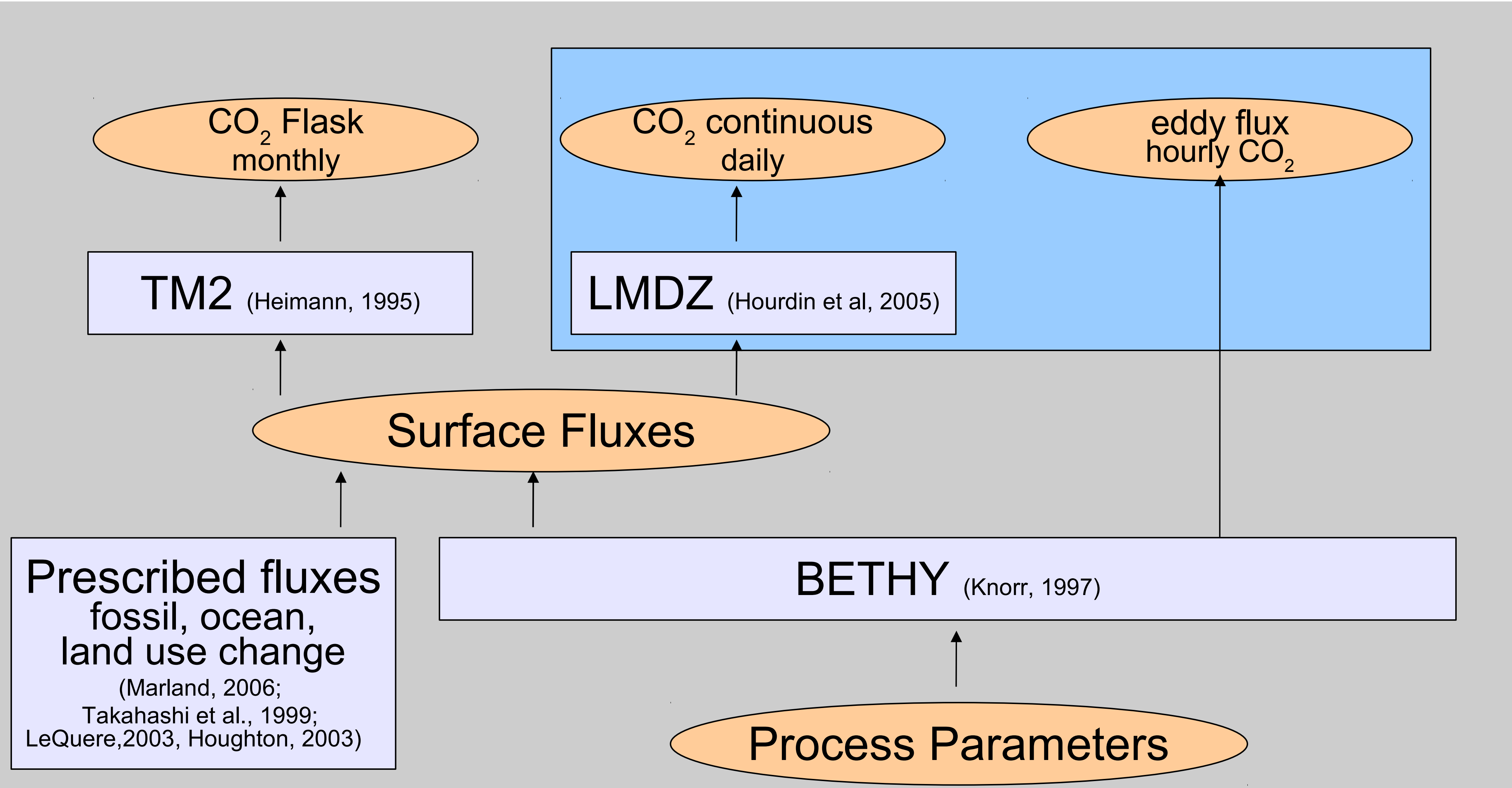
Rayner et al.
(Tellus, 1996)

Inverse model based on atmospheric tracer model

Existing network yields 1.16 GtC/yr

- Global constraints with narrow uncertainty on "ocean uptake + terrestrial uptake" couples information from disaparate regions:
  Improvement in knowledge of Amazonian fluxes improves knowledge of both total land and ocean fluxes!

Carbon Cycle Data Assimilation System (CCDAS) Forward Modelling Chain



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Model and Observational Uncertainties

- No observation/no model is perfect.
- It is convenient to quantify observations and their model counterpart by probability density functions (PDFs).
- The simplest assumption is that they are Gaussian.

$$\sigma_d^2 = \sigma_{obs}^2 + \sigma_{mod}^2$$

total uncertainty

uncertainty from model error

uncertainty from observational error

- If the observation refers to a point in space and time, there is a representation error because the counterpart simulated by the model refers to a box in space and time:
  the corresponding uncertainty must be accounted for either by the observational or by the model contribution to total uncertainty.

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Computing posterior parameter uncertainty

- M: x → d maps the process parameters onto the observations (CO_2 concentrations, flask and continuous, CO_2 hourly fluxes)
- Introduce cost function:

$$J(x) = \tfrac{1}{2} \left[(Mx-d)^T C_d^{-1} (Mx-d) + (x - x_{pr})^T C_{pr}^{-1} (x - x_{pr}) \right]$$

- x_{pr} : priori information on the process parameter
  C_d, C_{pr} : covariance matrices of uncertainties in observations and prior parameters respecively.

- If model M is **linear**, and if data d and priors x_{pr} have Gaussian PDF, then posterior PDF is also Gaussian (Tarantola 87):

$$\rho (x) = K * \exp(-\tfrac{1}{2}(x - x_{po})^T C_{po}^{-1}(x - x_{po}))$$

  with mean value:

$$x_{po} = x_{pr} + [M^T C_d^{-1} M + C_{pr}^{-1}]^{-1} M^T C_d^{-1} (d - M x_{pr})$$

- x_{po} minimises J(x)

- Uncertainty given by:

$$C_{po}^{-1} = M^T C_d^{-1} M + C_{pr}^{-1} = d^2 J(x_{po}) / dx^2$$

Computing posterior parameter uncertainty

- For **non-linear, differentiable** model M posterior distribution of parameters is still approximately Gaussian (Tarantola 87)
$$\rho (x) \approx K * \exp(-\tfrac{1}{2}(x - x_{po})^T C_{po}^{-1}(x - x_{po}))$$
- Equation for posterior uncertainties holds as approximation:
$$C_{po} \approx (d^2J(x_{po}) / dx^2)^{-1}$$
- x_{po} now determined by minimising cost function J
- Use gradient based optimisation to solve, x_{po} fullfills:
$$dJ(x_{po}) / dx = 0$$
- Efficient first and second order derivatives required, use adjoint code for frist order derivatives
- All derivative code generated from model code by automatic differentiation tool **TAF** (Giering and Kaminski, 1998)

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Network Desgin Tool: Simplifications of implementation

- For a number of i = 1,...,n_d of **uncorrelated** observations (C_d diagonal matrix) with (user provided) standard deviations σ_{d_i} the cost function can be written:

$$J(x) = ½ (x – x_{pr}) C_{pr}^{-1} (x – x_{pr}) + ½ \sum_{i=1}^{nd} ((M_i(x) – d_i)/\sigma_{d_i})^2$$

with:

$$C_{po}^{-1} = d^2J(x_{po}) / dx^2 = C_{pr}^{-1} + \sum_{i=1}^{nd} (1/\sigma_{d_i}^{2}) d^2/dx^2(M_i(x) – d_i)^2$$

uncertainies in observations **AND** model

- Using synthetic data d = M(x), model contribution in formula for C_{po} can be precomputed (do not depend on user input!)

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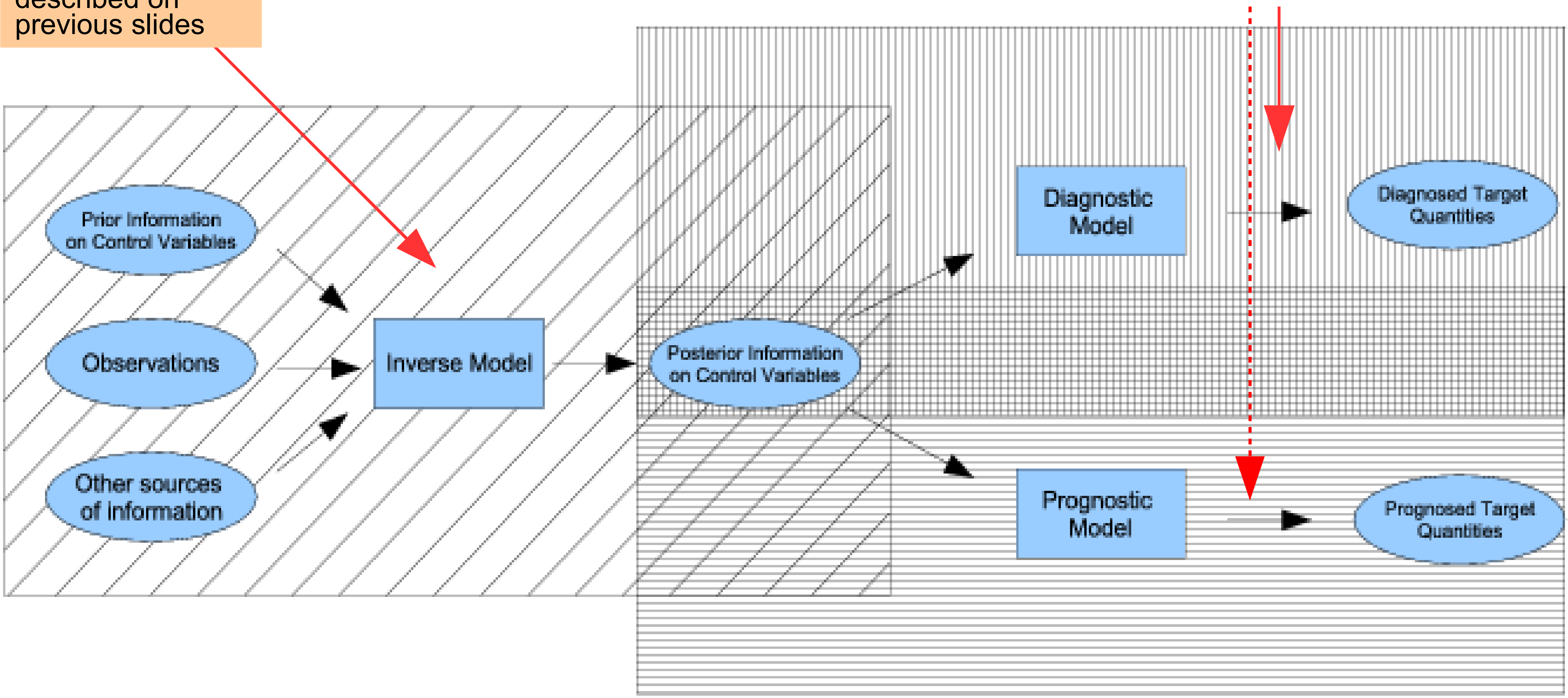
CCDAS scheme

Inversion step: described on previous slides

Propagation step: shown next



Propagation step

- N: x → y maps process parameters onto the target quanity
  (diagnostic or prognostic)
- Diagnostic run means that it is based on same domain (spatial and temporal) as used in the inversion step, whereas prognostic run is (at least in parts) outside this domain
- PDF of y can again be approximated by a Gaussian with mean
  $$y_{po} = N(x_{po})$$
  and covariance
  $$C(y_{po}) = D(N)\,C_{po}\,D(N)^T + C(y_{mod})$$
- $D(N) = dN(x_{po})/dx$ is the linearisation of N around x_{po} (Jacobian), $C(y_{mod})$ accounts for the uncertainty of the target quantity for errors in N
- Recall $C_{po}^{-1} = C_{pr}^{-1} + \sum_{i=1}^{nd} (1/\sigma_{d_i}^{2})\, d^2/dx^2(M_i(x) - d_i)^2$
- Answers the initial question and eventually yields mapping from uncertainties of (potential) observations to uncertainty of the target quantity:
  $C(y_{po}) = C(y_{po}; \sigma_{d_1}, \ldots, \sigma_{d_{ND}})$

Propagation step also requires derivatives!

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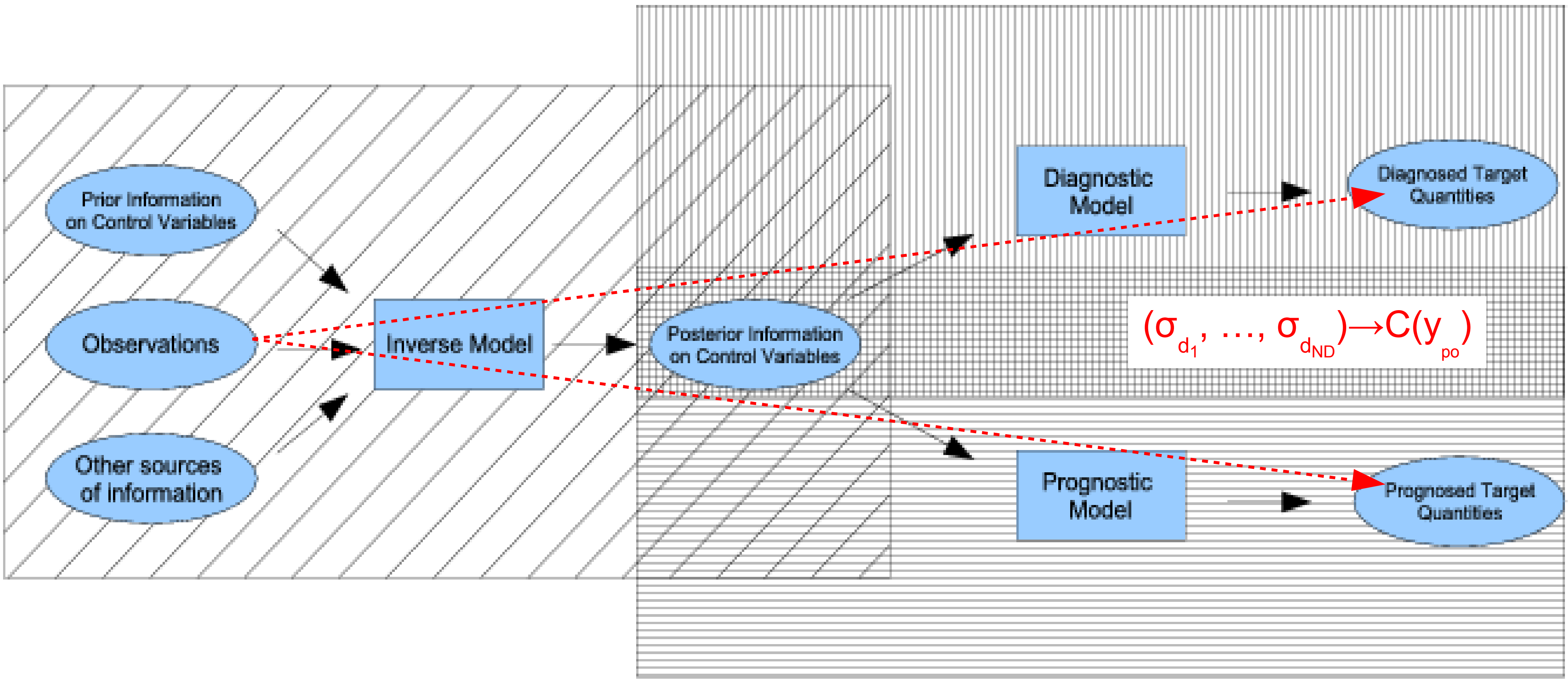
Uncertainties of Target Quantities



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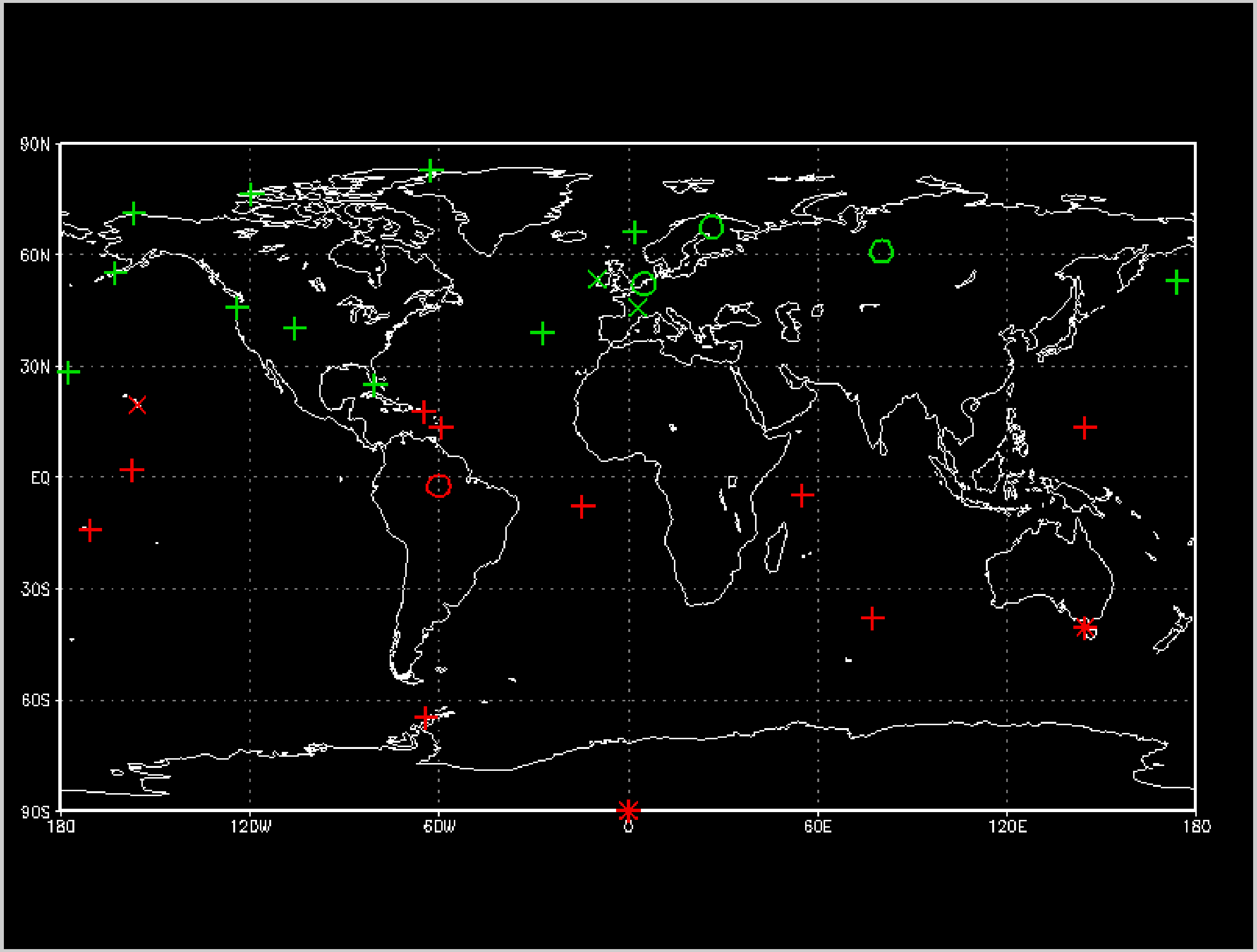
Sketch of Network DesignTool



Observations [sigma]

+ Flask [enter]
x Continuous [enter]
o Eddy Flux [enter]

|Compute|

Targets [sigma]

European Uptake [ ]
Global Uptake [ ]

Demo: Network Desgin Tool

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Summary: Assumptions and Ingredients

Assumptions:

- Gaussian uncertainties on priors, observations, and from model error (or function of Gaussian, e.g. lognormal)
- Model not too non linear

Ingredients:

- Assimilation system that can (efficiently) propagate uncertainties; helpful: Adjoint, Hessian, and Jacobian codes
- Ability to estimate uncertainties for priors, observations and due to model error; requires expertise of observationalists and modellers
- Need to take logistic constraints into account

Further Information

Terrestrial assimilation system applications and papers:
http://CCDAS.org

The corresponding network design project:

http://IMECC.CCDAS.org

with links to papers on network design

More assimilation systems, applications and papers:

http://FastOpt.com

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